Observations of Geographically Correlated Orbit Errors for TOPEX/POSEIDON Using the Global Positioning System

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Abstract, We have compared GPS-based dynamic and reduced-dynamic TOPEX/POSEIDON orbits over a 10-day period beginning March 10, 1993. The results suggest that the pre-launch gravity model (JGM-1) introduces geographically correlated errors which, when projected on a mean sea surface, have a strong meridional dependence, This dependence can be approximated by a large-scale positive anomaly in the Indian Ocean and a large-scale negative anomaly in the eastern Pacific Ocean. The global distribution and magnitude of these geographically correlated errors are consistent with pre-launch covariance analysis, with the estimated and predicted global rms error statistics agreeing exactly at 2.4 cm rms. Repeating the analysis with the post-launch gravity model (JGM-2) suggests that a portion of the meridional dependence observed in JGM-1 still remains, with a global rms of 1.4 cm, Such an anomaly has important implications on modeling the global circulation pattern of the world's oceans, which is the principal goal of TOPEX/POSEIDON. Assuming that JGM-2 represents an improvement in the earth's gravity model, the results presented here demonstrate that GPS reduced-dynamic orbits are relatively insensitive to the geographically correlated errors in JGM-1, Studies are underway to ascertain the origin of the residual geographically correlated errors present in differences between the GPS dynamic and reduced-dynamic JGM-2 orbits,

Introduction

The mean sea surface is an expression of both the gravity field of the earth and the large-scale circulation of its oceans. In the absence of systematic errors, measurements of the departure of the sea surface from the geoid can reveal the evolution of global circulation patterns as well as eustatic change in sea level. The goal of TOPEX/POSEIDON is to obtain such measurements using a complement of precision tracking and altimetric systems, In simple terms, the tracking and altimetric systems provide precise observations of the height of the satellite relative to the center of the earth and the height of the satellite relative to the sea surface, respectively, Properly combined, orbit and altimeter data provide an absolute measure of geocentric sea level.

The orbit enters into the measurement of the sea level in two important ways; it dictates the temporal and spatial sampling pattern of the altimeter, and it represents the reference to which the altimeter measurements are made, Therefore, our knowledge of the radial component of the orbit, as obtained through the process of precision orbit determination (POD), is of fundamental importance to TO PEX/POSEIDON. Owing to the relevance of POD to the mission, three tracking systems were adopted; Satellite Laser Ranging (SLR), Doppler Orbitography and Radio-positioning Integrated by Satellite (DORIS), and the Global Positioning System (GPS). Since the launch of TOPEX/POSEIDON on August 10,1992, a number of studies have been conducted to assess the scientific integrity of the tracking and altimetric systems, These include the evaluation of the global data sets as well as comparisons with in situ data at selected verification sites. This paper presents some of the early results obtained from a number of these studies, with a focus on the intercomparison of GPS dynamic orbits and GPS reduced-dynamic orbits [Yunck et al, this issue] in terms of geographically correlated errors.

Orbit Errors: Time Series

Figure 1 is a typical time series for the orbital height differences between two dynamic orbits, one produced with GPS data and the other with SLR and DORIS data for cycle 18, The JGM-2 gravity model [Nerem, et al, 1993(b)] was used in both cases. These differences result from any one or a number of dissimilarities *in* the dynamic models, tracking data types, tracking data distribution, coordinate systems, station locations, and filtering techniques used to determine the orbits, The time series can be roughly characterized as a modulated one cycle-per-revolution (cpr) signal, and reflects the type of orbit error that has plagued the recovery of sea level from altimetry from previous missions. Note however that the amplitude is quite small, with peak-to-peak variations of less than 12 cm. The one cycle-per-day (cpd) modulation is due in part to the daily updates made to the non-gravitational force model parameters during the orbit determination process [Chelton and Schlax, 1993], The root-mean-squared (rms) of the radial orbit differences is 2,7 cm which, when compared to the mission goal of 13 cm rms for the global distribution of radial orbit error [Born et al, 1984], represents excellent agreement,

The fact that orbit ephemerides compare well is a necessary, but not sufficient, condition for demonstrating their accuracy. If errors in the ephemerides were highly correlated, they would be unobservable in the orbit differences, Therefore, a proper interpretation of the differences between two orbits requires an understanding of both the random and systematic nature of various error sources, These error sources are discussed in the context of dynamic, geometric, and reduced-dynamic orbit determination techniques below.

Dynamic orbits, i.e. orbits that depend on dynamic models for propagation of the state, are subject to errors in the initial conditions and the models used to propagate the initial conditions forward in time. Initial condition error arises from tracking data noise, systematic errors in the tracking data, the distribution of tracking data, errors in the orientation and origin of the terrestrial reference frame (station locations), and errors in the dynamic models themselves, The model for the Earth's gravity field, as well as the radiative and drag force (non-gravitational) models for the spacecraft, play a significant role in the propagation of orbit error, These errors have a very rich spectrum with most of the energy at low wave number (<10'3 km^{-1}) , covering a fairly broad band centered at one cpr. Based on linear orbit perturbation theory [Kaula, 1966], all terms in the gravity model can give rise to orbital perturbations at or near one cpr, which accounts for the broadening of the spectrum at that frequency [Nerem et al, 1993(a)], This is also the dominant frequency for non-gravitational force model errors; however, the spectral line is much narrower than it is for gravity and, as discussed in the section above, splits into two spectral lines at (1 cpr + 1 cpd) and (1 cpr -1 cpd) due in part to the estimation of daily non-gravitational force parameters [see Chelton and Schlax, 1993],

Initial condition errors introduced by tracking system noise and uneven tracking data distributions will generally produce randomly phased position errors at discrete frequencies of one and perhaps two cpr [e.g. Engelis, I988]. For example, two orbits determined with different data types, say SLF? and GPS, are likely to have radial error components that are out of phase with each other. Mis-modeled non-gravitational forces can also introduce an element of randomness; in contrast, gravitational model and reference frame errors will introduce position errors that are highly correlated, even when the orbits are determined with different data sets,

Geometric orbits, i.e. orbits that depend on *only* the tracking metric, do not require dynamic models and are therefore subject only to errors associated with the tracking data, However, a purely geometric solution requires continuous, multidimensional observations. For TOPEX/POSEIDON, a hybrid of both geometric and dynamic methodologies has been used, referred to as the reduced-dynamic technique, where the geometric solution is loosely constrained to an *a priori* dynamic orbit [Yunck et al, this issue], It follows that an improved dynamic orbit enhances the performance of the reduced-dynamic technique. Insofar that reduced-dynamic GPS orbits are ostensibly geometric orbits, intercomparisons with GPS, SLR, and DORIS dynamic orbits can reveal deficiencies in the dynamic models and errors in the different tracking systems.

Such a comparison was done for TOPEX/POSEIDON 10-day-repeat cycle 18 covering the period from March 10 to March 20, 1993 using JGM-1, It was observed that the differences between the dynamic orbits and the GPS reduced-dynamic orbits (-3.9 cm rms) were generally larger than the difference between any two dynamic orbits (-2,7 cm rms). Assuming the reduced-dynamic orbits represent an improvement, this suggests that the dynamic orbits have highly correlated, albeit small, errors that are unobservable in the differences between them. In the next section, we demonstrate that the error is attributable in large part to the JGM-1 gravity model; moreover, that the reduced-dynamic orbits are relatively insensitive to the gravity model errors at the level represented by JGM-1 and JGM-2.

Orbit Errors: Geographical

For a repeat-track mission such as TOPEX/POSEIDON, time translates into geographical location in a cyclical pattern, Therefore, it is expected that time correlated error in the dynamic orbits are geographically correlated as well, Since the gravity field is fixed in the earth, orbit errors resulting from the gravity model are expected to be highly correlated with geographical location. Rosborough [1986] developed a comprehensive formulation for determining radial orbit perturbations due to the geopotential as a function of geographical location using linear theory. This formulation can be used to transform the error covariance matrix associated with a given gravity model into a geographical distribution of expected radial orbit error. Figure 2 shows such a transformation of the JGM-1 covariance. It is important to note that this represents the global distribution of the standard deviation for the radial orbit error, i.e. the statistical nature of the error, and thereby has no algebraic sign. An analysis of the cross-correlation between neighboring points show that these errors are highly correlated over large expanses of the earth's surface (c 104 km), i.e. if the error were positive in sign at the middle of the Pacific, the errors over a major portion of the Pacific would also be positive and vary in accordance with Figure 2. The most informative aspect of this description of orbit error is not so much the magnitude of the errors, which can be adjusted by scaling the error covariance matrix, but the geographical distribution of the errors. The pattern, as well as the magnitude, of the geographically correlated orbit error has important implications on modeling the global circulation pattern of the world's oceans.

Geographically correlated errors in the JGM-1 gravity model are evident in Figure 3, where the global distribution of the orbital height differences between the GPS reduced-dynamic and GPS dynamic ephemerides using JGM-1 for cycle 18 is shown. As noted in the previous section, the reduced-dynamic technique is somewhat sensitive to the dynamic orbit adopted as *a priori,* In that both JGM-1 and JGM-2 are highly accurate gravity models, the sensitivity is almost negligible, (The rms radial orbit difference between two reduced-dynamic orbits using *a priori* orbits generated with JGM-1 and JGM-2 is 0,47 cm for cycle 18,) Nonetheless, to ensure no preferential weighting of the reduced dynamic solution to the tuned JGM-2 model, JGM-1 was adopted for the *a priori* dynamic orbit. To first order, the orbital differences are geographically correlated and have a strong meridional dependence, This dependence can be approximated by a large-scale positive anomaly in the Indian Ocean and a large-scale negative anomaly in the eastern Pacific Ocean, It was observed that this anomaly persists for cycles 7, 10, and 14; therefore, it is not an ephemeral feature, The fact that the geographical distribution of these orbit differences so closely corresponds to the expected geographically correlated error pattern for JGM-1 strongly suggests that it has its origin in the gravity model (see Figure 2).

The most compelling evidence that this anomaly is attributable to an error in the JGM-1 gravity model is depicted by Figure 4, where the global distribution of the orbital height differences between two GPS dynamic orbits produced with the JGM-1 and JGM-2 gravity models is shown, Note that these differences come entirely from the gravity model. This figure is remarkably similar to Figure 3, with the exception that the meridional variation is smaller and there is less *trackiness,* i.e. less disparity among neighboring tracks. The JGM-2 gravity model is basically the JGM-1 gravity model *tuned* with TOPEX/POSEIDON SLR and DORIS data from cycles 1-15 from September 20,1992 through February 18, 1993, It is important to note that no GPS data was used to obtain JGM-2, i.e. it is highly unlikely that a geographically correlated error in the GPS tracking system would appear as an alias in JGM-2. Assuming that JGM-2 represents an improvement, these results suggest that GPS reduced-dynamic orbits have recovered from some of the geographically correlated error in JGM-1,

To further quantify this conclusion, we have adopted spherical harmonics to discriminate the geographically correlated error from the tackiness. Using the same cycle 18 comparisons, differences between the dynamic and reduced-dynamic orbit heights were projected onto the mean sea surface using a least-squares process to obtain a tenth order and degree spherical harmonic representation of the data, This was done for each of the ascending, descending, and ensemble global data sets. It was observed that most of the energy was absorbed by the low order terms (n c 4) and that n = 10 was adequate for establishing the noise-floor.

In interpreting the results, it is instructive to first consider the following definitions, The rms geographically correlated orbit error associated with a global mean surface derived by averaging ascending and descending passes of altimeter data will be referred to as the *Mean GCE.* The rms geographically correlated error associated with the global ensemble of ascending and descending passes will be referred to as the *Ascending GCE* and *Descending GCE* respectively, and the weighted average of these statistics as the Tots/ *GCE.* The correlation between the orbit error for the ascending pass and the descending pass at a given geographical location, known as a crossing-point, accounts for the distinction between *Tots/ GCE* and *Mean GCE.* The *Tots/ GCE, Ascending GCE,* and *Descending GCE* are unaffected by correlations at the crossing-points and, since the gravity errors are not preferentially larger for the ascending or descending global data sets, are approximately equal, The *Mean GCE* will tend toward O if the ascending and descending passes are negatively correlated and will tend toward the *Tots/ GCE* if they are positively correlated, Rosborough [1986] has shown that the *Mean* GCEfor the early gravity models for the Earth was approximately $2^{-1/2}$ times the *Tots/ GCE,* which indicates that errors associated with the ascending and descending passes are relatively uncorrelated.

Table 1 lists the statistics for the various spherical harmonic fits performed on the cycle 18 orbit differences. In the first case, we consider the two dynamic orbits based on JGM-1 and JGM-2. Inasmuch as the only difference between the two solutions are the gravity models, this comparison serves as an ideal means of demonstrating the ability of the spherical harmonics to model the geographically correlated orbit error. When the ascending and descending passes are considered separately, the spherical harmonics are very effective at reducing the differences, absorbing more than 80'% of the total energy. For the mean surface, the reduction is less dramatic, This suggests that, in a global sense, there is some decorrelation of the error along the ascending and descending tracks,

The next case considers the difference between the JGM-1 dynamic and reduced-dynamic orbits, In this comparison, the rms of the spherical harmonic surface field represents our estimate of the *Mean GCE* introduced by JGM-1, The numbers are lower than the rms of the along-track radial orbit differences because we have effectively filtered out the *tackiness* seen in Figures 3 and 5 by employing spherical harmonics to discriminate the gravity-induced errors from the time-dependent errors. Note that the residual rms accounts for the combined effect of errors in the non-conservative force models, initial conditions, and time-dependent errors in the reduced-dynamic orbits, (In the case of the *Mean GCE*, the residual rms also contains the portion of the gravity-induced errors that are, in a global sense, decorrelated at the crossing-points.) When the comparison is repeated for JGM-2, a marked reduction in the energy of the geographically correlated error signal is observed (from 2.4 cm to 1,4 cm rms),

Further evaluations of our techniques are provided in Table 2, where we have compiled our estimates of the TOPEX,/POSEIDON geographically correlated error along with predictions based on the applications of linear theory to the error covariance matrix. For JGM-1, our estimate of the *Mean GCE* (2,4 cm) corresponds exactly to the pre-launch prediction (cf. Figure 2), Likewise, our estimate of the Tots/ *GCE* (3.2 cm) is in accord with the corresponding prediction of the total gravity-induced error for the TOPEX/POSEIDON orbit (3.4 cm). For JGM-2, the agreement is also very close, with both techniques suggesting that the gravity-induced errors have indeed been reduced to the level of 2 cm rms. These results not only provide a tangible demonstration of the unprecedented strength of the GPS data from TOPEX./POSEIDON, they also serve as evidence of the remarkable achievements in the gravity modeling effort,

Comments

The results presented here suggest that the GPS reduced-dynamic technique has enormous potential for reducing the error in the orbit of TOPEX/POSEIDON, even in the presence of dynamic model errors, Both GPS reduced-dynamic and classical dynamic orbit determination methodologies are capable of observing geographically correlated errors such as those introduced by the pre-launch JGM-1 gravity model. Though JGM-2 is definitely an improvement over JGM-1, a measurable amount of energy remains in the differences between reduced-dynamic and dynamic orbits, More extensive comparisons between dynamic and geometric orbits will be necessary to isolate the sources of this error.

TOPEX/POSEIDON has been used to demonstrate that the GPS reduced-dynamic technique is capable of recovering from significant gravitational and non-gravitational modeling errors. It follows that, when used with dynamic orbit determination techniques, GPS data is capable of improving the Earth's gravity model (Shutz et al, this issue), Once such an improved model is obtained, the GPS dynamic orbits are expected to be even more comparable to GPS reduced-dynamic orbits for TOPEX/POSEIDON. This may not necessarily be the case for low Earth orbiters, where non-gravitational forces are much more difficult to model. Therefore, the GPS reduced-dynamic technique will be particularly important for low-altitude altimeter missions where errors due to atmospheric drag are especially large.

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References

Born, G.H., C. Wunsch, and C.A. Yamarone, TOPEX: Observing the oceans from space, *EOS Trans., AGU,* 65, 433-434, 1984,

Chelton, D.B. and M.G. Schlax, Spectral characteristics of time-dependent orbit errors in altimeter height measurements, *J. Geophys. Res.*, accepted February, l993.

Engelis, T., On the simultaneous improvement of a satellite orbit and determination of sea surface topography using altimeter data, *manuscripta geodaetica*, 13, 180-190, [988.

Rosborough, G,W., Satellite orbit perturbations due to the geopotential, CSR-86-1, Center for Space Research, The University of Texas at Austin, l986.

Nerem, R, S., B.H. Putney, J.A. Marshall, F.J. Lerch, E.C. Pavlis, S.M. Klosko, S.B. Luthcke, G.B. Patel, R.G. Williamson, and N.P. Zelensky, Expected orbit determination performance for the TOPEX/POSEIDON mission, */EEE Trans. Geosci. and Rem, Sens., 31, 2, 333-354,* 1993(a).

Nerem, R. S., et al, Gravity model improvement for TOPEX/POSEIDON, *J. Geophys. Res.*, pending, 1993(b).

Figure Captions

Fig. 1. Time series of the differences in the radial component of a dynamic orbit determined with GPSDR data and a dynamic orbit determined with SLR and DORIS for cycle 18 of TOPEX/POSEIDON, both generated with the JGM-2 gravity model.

Fig. 2, Geographically correlated radial orbit error for TOPEX/POSEIDON predicted by the JGM-1 error covariance.

Fig, 3. Global distribution of the orbital height differences between the GPS dynamic and GPS reduced-dynamic ephemerides using JGM-1 for cycle 18 of TOPEX/POSEIDON.

Fig, 4. Global distribution of orbital height differences between a GPS dynamic orbit generated with the JGM-1 gravity model and a GPS dynamic orbit generated with JGM-2 gravity model,

Fig, 5. Global distribution of the orbital height differences between the GPS dynamic and GPS reduced-dynamic ephemerides using JGM-2 for cycle 18 of TOPEX/POSEIDON.

Orbits compared	Method	RMS of GCE (cm)[1]	Residual RMS (cm)
JGM- 1- JGM-2	Ascending	1.64	0.59
	Descending	1.62	0.64
	Mean	1.38	1.08
JGM-1 – Reduced	Ascending	3.25	2.26
	Descending	3.15	2.13
	Mean	2.40	3,05
JGM-2 – Reduced	Ascending	1.89	2.00
	Descending	2.14	1.84
	Mean	1.36	2.43

1 GCE: geographically correlated error

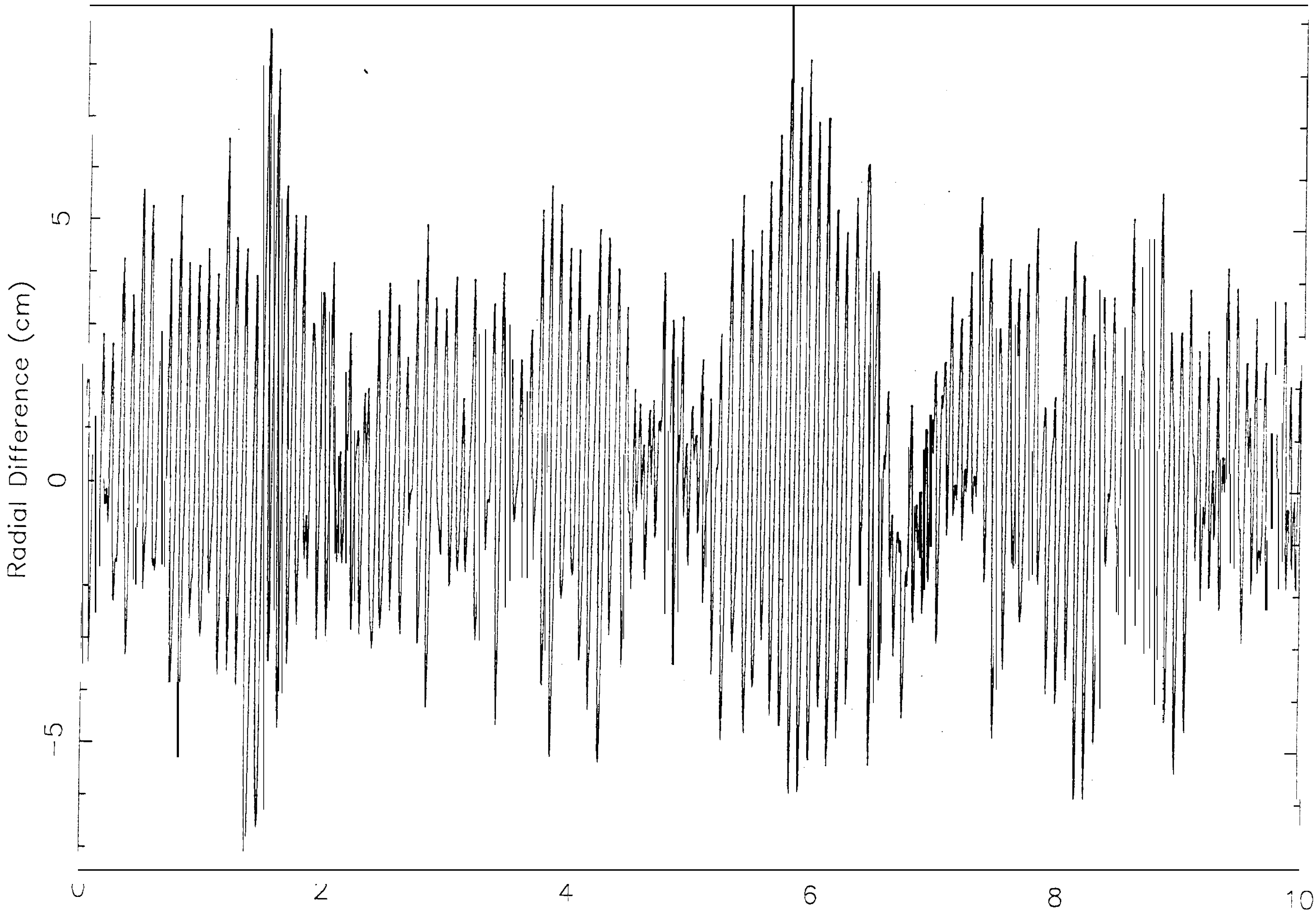
Table 1. Topex/Poseidon geographically correlated orbit errors from 10 X 10 spherical harmonic (SH) expansion of orbit differences. The residual RMS is the error remaining in the orbit differences after removal of the SH functions.

Gravity Field	GCE Type	Actual RMS (cm)	Predicted RMS (cm)
JGM-1	Total[1]	3.2	3.4
	Mean[2]	2.4	2.4
JGM-2	Total[1]	1.9	2.2
	Mean[2]	1.4	1.6

1 Total GCE observed along satellite track.
2 Mean GCE (observed in average of ascending and descending tracks).

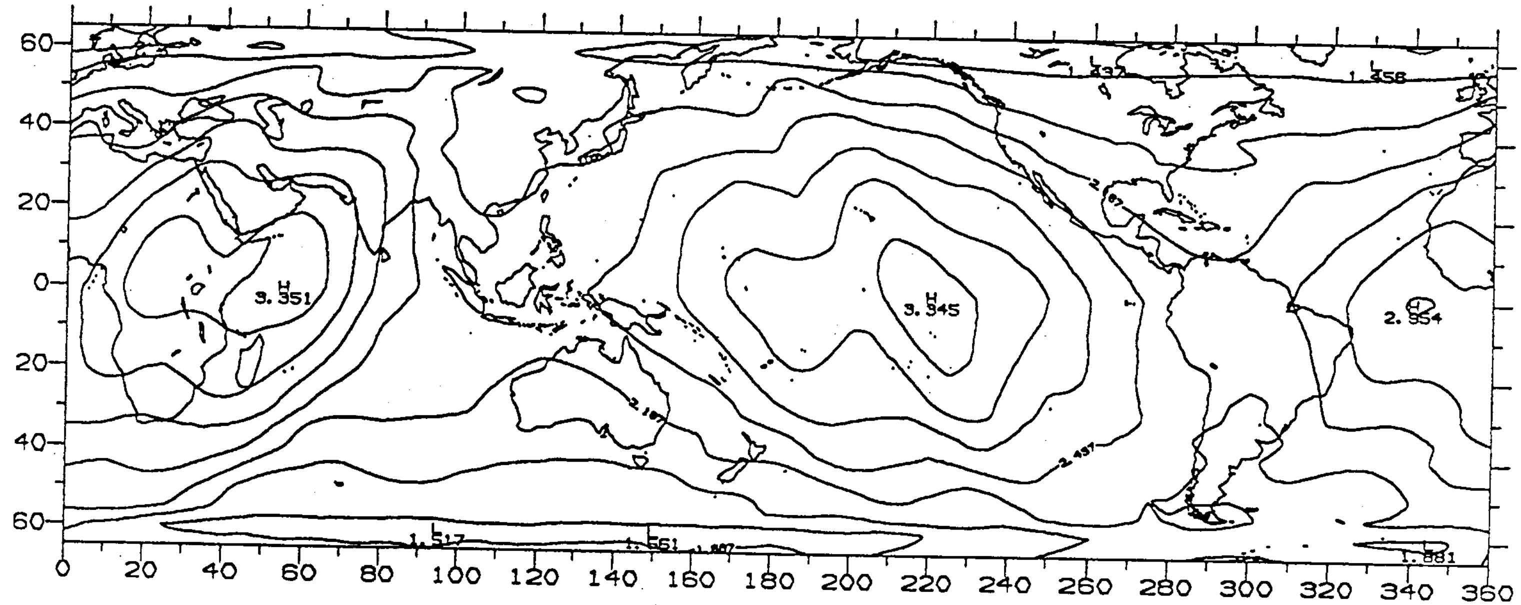
Table 2. Topex/Poseidon geographically correlated orbit errors based on comparison wit h the GPS reduced dynamic orbits (actual) and application of linear theory to the full gravity field covariance (predicted).

JGM 2 Dynamic Orbit Comparison GSFC SLR/Doris vs JPL GPS)



:GURE

GEOGRAPHICALLY CORRELATED RADIAL ORBIT ERROR
FOR TOPEX PREDICTED BY JGM-1 GRAVITY COVARIANCE



CONTOUR INTERVAL = 0.250 C M

MINIMUM = 1.44 MAXIMUM = 3.35

MEAN = 2.35 RMS = 2.39

FIGURE 2

Cycle 18 Radial Orbit Differences: JGM- Dynamic - Reduced Dynamic

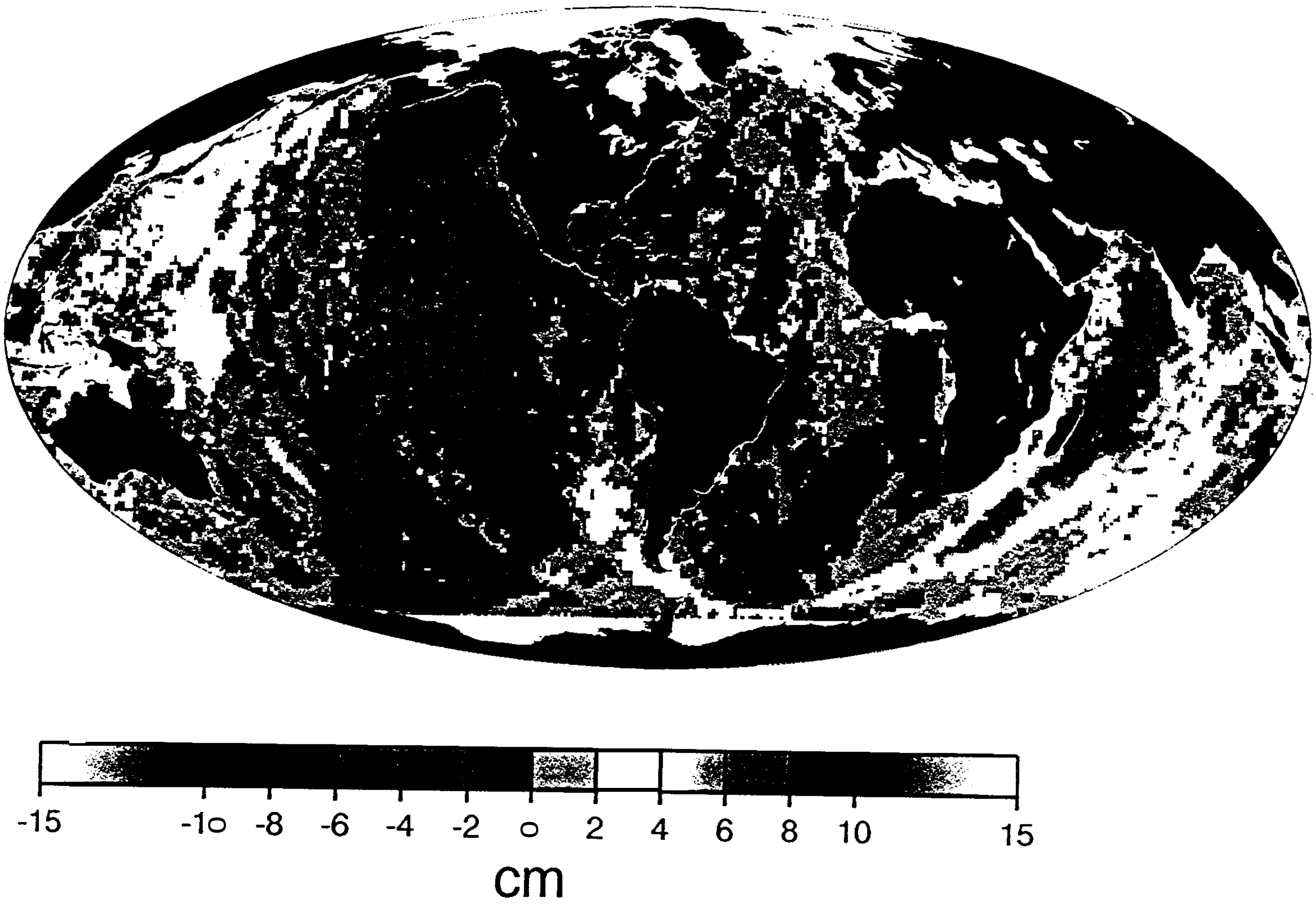


FIGURE 5

Cycle 18 Orbit Differences: JGM-   Dynamic - JGM-2 Dynamic

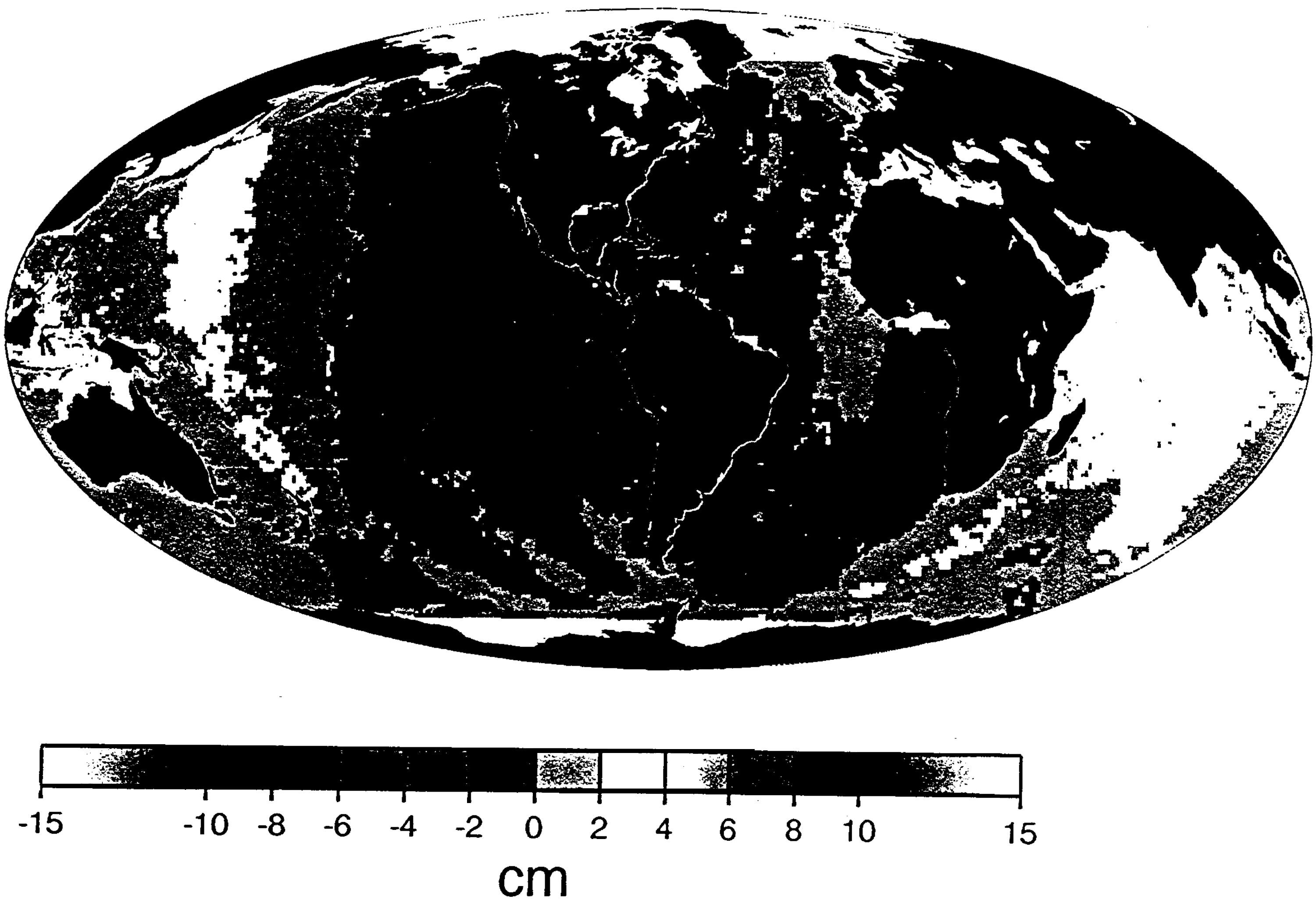


FIGURE 4

Cycle 18 Radia Orbit Differences: JGM-2 Dynamic - Reduced Dynamic

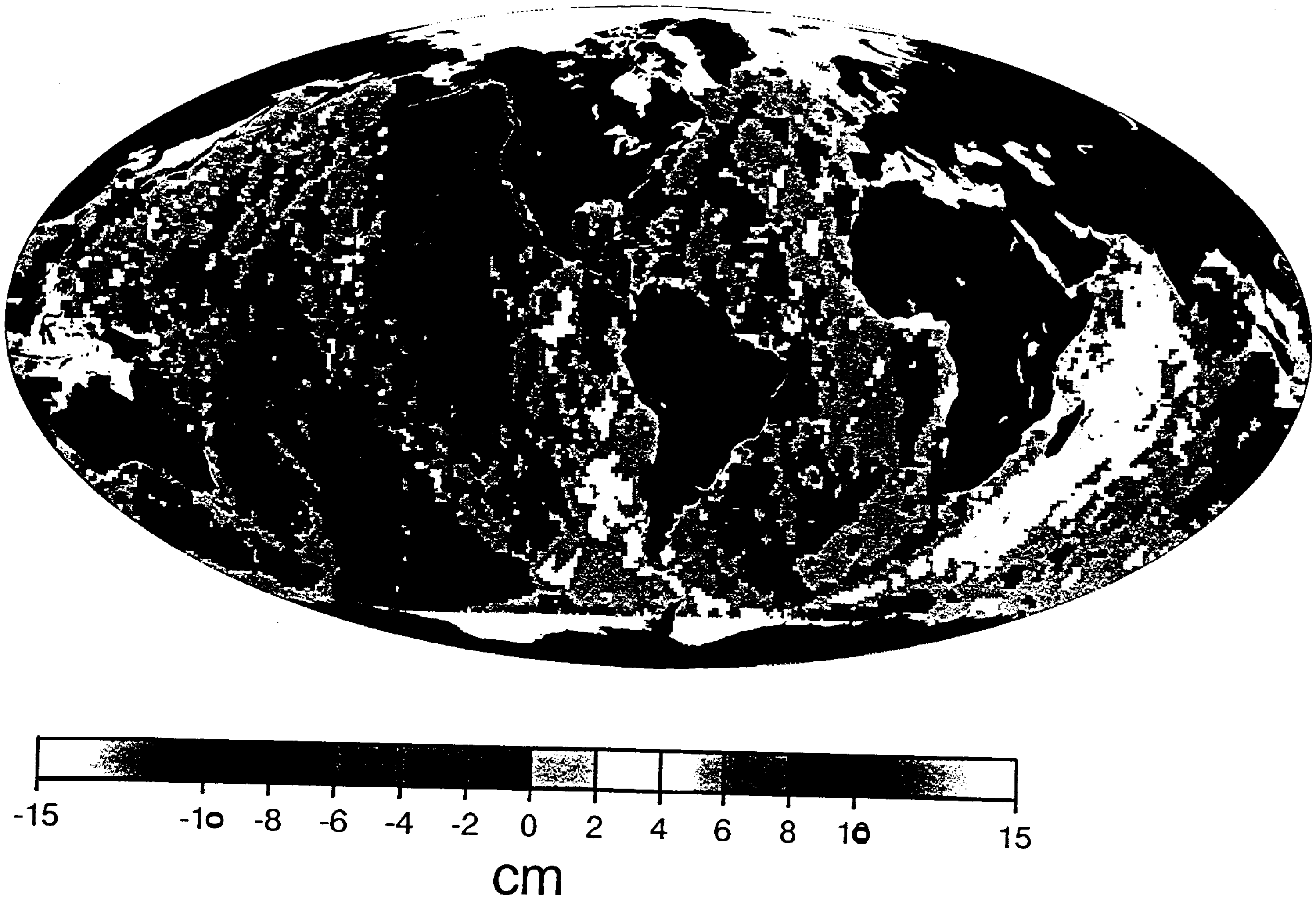


FIGURE 5